Perpetual Debt Valuation
Revenue Growth Rate and Debt Ratio Are Constants

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In this white paper we will build a model to value perpetual debt assuming a constant (1) revenue growth rate and (2) ratio of debt to annualized revenue. To that end we will work through the following hypothetical problem...

Our Hypothetical Problem

We are currently standing at time zero and are tasked with determining the market value of a company's interest-bearing debt. Our go-forward model assumptions are...

Description	Value
Annualized revenue (in dollars)	1,000,000
Annualized revenue growth rate (%)	3.50
Ratio of interest-bearing debt to annualized revenue	0.35
Ratio of non interest-bearing debt to annualized revenue	0.20
Debt market yield (%)	6.00
Risk-adjusted discount rate (%)	9.50

Question 1: What is the book value of ib debt and nib debt at time zero?

Question 2: What is the market value of ib debt at time zero assuming that the debt coupon rate is 5.25%?

Question 3: What is the market value of ib debt at time zero assuming that the debt coupon rate is 6.50%?

Question 4: What is the market value of ib debt at time zero assuming that the the debt coupon rate is 6.00%?

Question 5: What is the market value of nib debt at time zero?

Question 6: What is the market value of the debt tax shield at time zero assuming that the the debt coupon rate is 6.00%?

Balance Sheet

We will define the variable R_t to be annualized revenue at time t and the variable ω to be the continuous-time revenue growth rate. The equation for annualized revenue at time t as a function of annualized revenue at time zero is...

$$R_t = R_0 \operatorname{Exp}\left\{\omega\, t\right\} \text{ ...where... } \frac{\delta R_t}{\delta t} = \omega\, R_0 \operatorname{Exp}\left\{\omega\, t\right\} \text{ ...such that... } \delta R_t = \omega\, R_0 \operatorname{Exp}\left\{\omega\, t\right\} \delta t \tag{1}$$

We will define the variable D_t to be the interest-bearing debt balance at time t and the variable ϵ to be the ratio of interest-bearing debt to annualized revenue. Using Equation (1) above, the equation for interest-bearing debt balance at time t is...

$$D_t = \epsilon\, R_t = \epsilon\, R_0 \operatorname{Exp}\left\{\omega\, t\right\} \text{ ...where... } \frac{\delta D_t}{\delta t} = \omega\, \epsilon\, R_0 \operatorname{Exp}\left\{\omega\, t\right\} \text{ ...such that... } \delta D_t = \omega\, \epsilon\, R_0 \operatorname{Exp}\left\{\omega\, t\right\} \delta t \tag{2}$$

We will define the variable G_t to be the non interest-bearing debt balance at time t and the variable η to be the ratio of non interest-bearing debt to annualized revenue. Using Equation (1) above, the equation for non interest-bearing debt balance at time t is...

$$G_t = \eta\, R_t = \eta\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \text{ ...where... } \frac{\delta G_t}{\delta t} = \omega\, \eta\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \text{ ...such that... } \delta G_t = \omega\, \eta\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{3}$$

Increase/(Decrease) in Debt

We will define the variable P_t to be the increase/(decrease) in interest-bearing debt over the infinitesimally small time interval $[t, t+\delta t]$. Using Equation (1) above, the equation for the change in interest-bearing debt at time t is...

$$P_t = \epsilon\, \delta R_t\, \delta t = \omega\, \epsilon\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{4}$$

We will define the variable $P_{a,b}$ to be the cumulative increase/(decrease) in interest-bearing debt over the time interval $[a, b]$. Using Equation (4) above, the equation for the cumulative change in interest-bearing debt is...

$$P_{a,b} = \int\limits_a^b \omega\, \epsilon\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t = \omega\, \epsilon\, R_0 \int\limits_a^b \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{5}$$

Using Appendix Equation (38) and (39) below, the solution to Equation (5) above is...

$$P_{a,b} = \omega\, \epsilon\, R_0 \left[\mathrm{Exp}\left\{\omega\, b\right\} - \mathrm{Exp}\left\{\omega\, a\right\}\right] \omega^{-1} = \epsilon\, R_0 \left[\mathrm{Exp}\left\{\omega\, b\right\} - \mathrm{Exp}\left\{\omega\, a\right\}\right] \tag{6}$$

We will define the variable H_t to be the increase/(decrease) in non interest-bearing debt over the infinitesimally small time interval $[t, t+\delta t]$. Using Equation (1) above, the equation for the change in non interest-bearing debt at time t is...

$$H_t = \eta\, \delta R_t\, \delta t = \omega\, \eta\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{7}$$

We will define the variable $H_{a,b}$ to be the cumulative increase/(decrease) in non interest-bearing debt over the time interval $[a, b]$. Using Equation (7) above, the equation for the cumulative change in non interest-bearing debt is...

$$H_{a,b} = \int\limits_a^b \omega\, \eta\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t = \omega\, \eta\, R_0 \int\limits_a^b \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{8}$$

Using Appendix Equation (38) and (39) below, the solution to Equation (8) above is...

$$H_{a,b} = \omega\, \eta\, R_0 \left[\mathrm{Exp}\left\{\omega\, b\right\} - \mathrm{Exp}\left\{\omega\, a\right\}\right] \omega^{-1} = \eta\, R_0 \left[\mathrm{Exp}\left\{\omega\, b\right\} - \mathrm{Exp}\left\{\omega\, a\right\}\right] \tag{9}$$

Debt Service

We will define the variable C_t to be debt service payments made over the infinitesimally small time interval $[t, t+\delta t]$ and the variable α to be the debt service coupon rate. Using Equation (2) above, the equation for the debt service payment at time t is...

$$C_t = \alpha\, D_t\, \delta t = \alpha\, \epsilon\, R_t\, \delta t = \alpha\, \epsilon\, R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t \tag{10}$$

We will define the variable $C_{a,b}$ to be cumulative debt service payments made over the time interval $[a, b]$. Using Equation (10) above, the equations for cumulative debt service are...

$$C_{a,b} = \int\limits_a^b \alpha\, \epsilon R_0\, \mathrm{Exp}\left\{\omega\, t\right\} \delta t = \alpha\, \epsilon\, R_0 \int\limits_a^b \mathrm{Exp}\left\{\omega\, t\right\} \delta t \text{ ...when... } \omega \neq 0 \tag{11}$$

$$C_{a,b} = \int\limits_a^b \alpha\, \epsilon\, R_0\, \delta t = \alpha\, \epsilon\, R_0 \int\limits_a^b \delta t \text{ ...when... } \omega = 0 \tag{12}$$

Using Appendix Equations (38) and (39) below, the solutions to Equations (11) and (12) above are...

$$C_{a,b} = \alpha\,\epsilon\,R_0 \left[\text{Exp}\left\{\omega\,b\right\} - \text{Exp}\left\{\omega\,a\right\}\right]\omega^{-1} \text{ ...when... } \omega \neq 0 \tag{13}$$

$$C_{a,b} = \alpha\,\epsilon\,R_0 \left(b - a\right) \text{ ...when... } \omega = 0 \tag{14}$$

Debt Tax Shield

We will define the variable S_t to be the nominal value of the debt tax shield over the infinitesimally small time interval $[t, t+\delta t]$ and the variable τ to be the income tax rate. Using Equation (10) above as our guide, the equation for the value of the debt tax shield at time t is...

$$S_t = \alpha\,\tau\,D_t\,\delta t = \alpha\,\epsilon\,\tau\,R_0\,\text{Exp}\left\{\omega\,t\right\}\delta t \tag{15}$$

We will define the variable $S_{a,b}$ to be the nominal value of the debt tax shield over the time interval $[a, b]$. Using Equation (15) above, the equation for the cumulative debt tax shield is...

$$S_{a,b} = \int\limits_a^b \alpha\,\epsilon\,\tau\,R_0\,\text{Exp}\left\{\omega\,t\right\}\delta t = \alpha\,\epsilon\,\tau\,R_0 \int\limits_a^b \text{Exp}\left\{\omega\,t\right\}\delta t \text{ ...when... } \omega \neq 0 \tag{16}$$

$$S_{a,b} = \int\limits_a^b \alpha\,\epsilon\,\tau\,R_0\,\delta t = \alpha\,\epsilon\,\tau\,R_0 \int\limits_a^b \delta t \text{ ...when... } \omega = 0 \tag{17}$$

Using Equations (13) and (14) as our guide, the solution to Equations (16) and (17) above are...

$$S_{a,b} = \alpha\,\epsilon\,\tau\,R_0 \left[\text{Exp}\left\{\omega\,b\right\} - \text{Exp}\left\{\omega\,a\right\}\right]\omega^{-1} \text{ ...when... } \omega \neq 0 \tag{18}$$

$$S_{a,b} = \alpha\,\epsilon\,\tau\,R_0 \left(b - a\right) \text{ ...when... } \omega = 0 \tag{19}$$

Valuation

We will define the variable $V(IB)$ to be the value of perpetual interest-bearing debt at time zero and the variable ι to be the market yield on interest-bearing debt. Using Equations (4) and (10) above, the equation for the value of perpetual interest-bearing debt at time zero is...

$$V(IB) = \int\limits_0^\infty \left(C_t - P_t\right)\text{Exp}\left\{-\,\iota\,t\right\}\delta t = \int\limits_0^\infty (\alpha - \omega)\,\epsilon\,R_0\,\text{Exp}\left\{\omega\,t\right\}\text{Exp}\left\{-\,\iota\,t\right\}\delta t \tag{20}$$

Using Appendix Equation (44) below, the solution to Equation (20) above is...

$$V(IB) = \frac{\alpha - \omega}{\iota - \omega}\,\epsilon\,R_0 \text{ ...when... } \omega < \kappa \tag{21}$$

Note that when the debt service rate (α) equals the debt market yield (ι) the market value of interest-bearing perpetual debt equals the book value of interest-bearing perpetual debt at time zero.

We will define the variable $V(NIB)$ to be the value of perpetual non interest-bearing debt at time zero and the variable κ to be the risk-adjusted discount rate. Using Equation (7) above, the equation for the value of perpetual non interest-bearing debt at time zero is...

$$V(NIB) = \int\limits_0^\infty -H_t\,\text{Exp}\left\{-\,\kappa\,t\right\}\delta t = \int\limits_0^\infty -\omega\,\eta\,R_0\,\text{Exp}\left\{\omega\,t\right\}\text{Exp}\left\{-\,\kappa\,t\right\}\delta t \tag{22}$$

Using Appendix Equation (44) below, the solution to Equation (22) above is...

$$V(NIB) = -\frac{\omega}{\kappa - \omega}\,\eta\,R_0 \;\text{ ...when... }\; \omega < \kappa \tag{23}$$

We will define the variable $V(DTS)$ to be the value of the debt tax shield at time zero and the variable κ to be the risk-adjusted discount rate. Using Equation (15) above, the equation for the value of the debt tax shield at time zero is...

$$V(DTS) = \int\limits_0^\infty S_t\,\mathrm{Exp}\left\{-\kappa\,t\right\}\delta t = \int\limits_0^\infty \alpha\,\epsilon\,\tau\,R_0\,\mathrm{Exp}\left\{\omega\,t\right\}\mathrm{Exp}\left\{-\kappa\,t\right\}\delta t \tag{24}$$

Using Appendix Equation (44) below, the solution to Equation (24) above is...

$$V(DTS) = \frac{\alpha\,\tau}{\kappa - \omega}\,\epsilon\,R_0 \;\text{ ...when... }\; \omega < \kappa \tag{25}$$

Answers To Our Hypothetical Problem

Question 1: What is the book value of ib debt and nib debt at time zero?

Per our model go-forward assumptions, the values of R_0 (annualized revenue at time zero) and ϵ (ratio of debt principal to annualized revenue) are...

$$R_0 = 1{,}000{,}000 \;\text{ ...and... }\; \epsilon = 0.3500 \;\text{ ...and... }\; \eta = 0.2000 \tag{26}$$

Using Equations (2) and (26) above, the answer to the question is...

$$D_0 = 0.35 \times 1{,}000{,}000 = 350{,}000 \;\text{ ...and... }\; G_0 = 0.20 \times 1{,}000{,}000 = 200{,}000 \tag{27}$$

Question 2: What is the market value of debt at time zero assuming that the debt coupon rate is 5.25%?

The values of α (debt coupon rate), κ (market yield), and ω (continuous-time revenue growth rate) are...

$$\alpha = 0.0525 \;\text{ ...and... }\; \kappa = 0.0600 \;\text{ ...and... }\; \omega = \ln\left(1 + 0.0350\right) = 0.0344 \tag{28}$$

Using Equations (21), (26) and (28) above, the answer to the question is...

$$V_0 = \frac{0.0525 - 0.0344}{0.0600 - 0.0344} \times 0.35 \times 1{,}000{,}000 = 247{,}460 \tag{29}$$

Question 3: What is the market value of debt at time zero assuming that the debt coupon rate is 6.50%?

The values of α (debt coupon rate), κ (market yield), and ω (continuous-time revenue growth rate) are...

$$\alpha = 0.0650 \;\text{ ...and... }\; \kappa = 0.0600 \;\text{ ...and... }\; \omega = \ln\left(1 + 0.0350\right) = 0.0344 \tag{30}$$

Using Equations (21), (26) and (30) above, the answer to the question is...

$$V_0 = \frac{0.0650 - 0.0344}{0.0600 - 0.0344} \times 0.35 \times 1{,}000{,}000 = 418{,}359 \tag{31}$$

Question 4: What is the market value of debt at time zero assuming that the the debt coupon rate is 6.00%?

The values of α (debt coupon rate), κ (market yield), and ω (continuous-time revenue growth rate) are...

$$\alpha = 0.0600 \;\text{ ...and... }\; \kappa = 0.0600 \;\text{ ...and... }\; \omega = \ln\left(1 + 0.0350\right) = 0.0344 \tag{32}$$

Using Equations (21), (26) and (32) above, the answer to the question is...

$$V_0 = \frac{0.0600 - 0.0344}{0.0600 - 0.0344} \times 0.35 \times 1{,}000{,}000 = 350{,}000 \;\text{ ...when... }\; \omega < \kappa \tag{33}$$

Note that when the coupon rate equals the discount rate (market yield) the market value of the debt equals the book value debt.

Question 5: What is the market value of nib debt at time zero? The values of κ (discount rate) and ω (continuous-time revenue growth rate) are...

$$\kappa = \ln\left(1 + 0.0950\right) = 0.0908 \text{ ...and... } \omega = \ln\left(1 + 0.0350\right) = 0.0344 \tag{34}$$

Using Equations (23), (26) and (34) above, the answer to the question is...

$$V(NIB) = -\frac{0.0344}{0.0908 - 0.0344} \times 0.20 \times 1,000,000 = -121,986 \text{ ...when... } \omega < \kappa \tag{35}$$

Question 6: What is the market value of the debt tax shield at time zero assuming that the the debt coupon rate is 6.00%?

The values of α (debt coupon rate), κ (discount rate), τ (income tax rate) and ω (continuous-time revenue growth rate) are...

$$\alpha = 0.0600 \text{ ...and... } \kappa = \ln\left(1 + 0.0950\right) = 0.0908 \text{ ...and... } \tau = 0.1250 \text{ ...and... } \omega = \ln\left(1 + 0.0350\right) = 0.0344 \tag{36}$$

Using Equations (25), (26) and (36) above, the answer to the question is...

$$V(DTS) = \frac{0.0600 \times 0.1250}{0.0908 - 0.0344} \times 0.35 \times 1,000,000 = 46,543 \text{ ...when... } \omega < \kappa \tag{37}$$

Appendix

A. The solution to the following integral is..

$$I = \int\limits_a^b \text{Exp}\left\{x\,t\right\}\delta t = \frac{1}{x}\left[\text{Exp}\left\{x\,t\right\}\right]_a^b = \frac{1}{x}\left[\text{Exp}\left\{x \times b\right\} - \text{Exp}\left\{x \times a\right\}\right] \text{ ...when... } x \neq 0 \tag{38}$$

$$I = \int\limits_a^b \text{Exp}\left\{x\,t\right\}\delta t = \int\limits_a^b \delta t = b - a \text{ ...when... } x = 0 \tag{39}$$

B. The solution to the integrals in Equations (38) and (39) above when the bounds of integration are $[0, T]$ is...

$$I = \int\limits_0^T \text{Exp}\left\{x\,t\right\}\delta t = \frac{1}{x}\left[\text{Exp}\left\{x \times T\right\} - \text{Exp}\left\{x \times 0\right\}\right] = \frac{1}{x}\left[\text{Exp}\left\{x \times T\right\} - 1\right] \text{ ...when... } x \neq 0 \tag{40}$$

$$I = \int\limits_0^T \text{Exp}\left\{x\,t\right\}\delta t = T - 0 = T \text{ ...when... } x = 0 \tag{41}$$

C. Using Equation (40) above, the solution to the integral in Equation (38) above when the bounds of integration are $[0, \infty]$ and $x < 0$ is...

$$I = \int\limits_0^\infty \text{Exp}\left\{x\,t\right\}\delta t = \frac{1}{x}\left[\text{Exp}\left\{x \times \infty\right\} - \text{Exp}\left\{x \times 0\right\}\right] = \frac{1}{x}\left[0 - 1\right] = -\frac{1}{x} \text{ ...when... } x < 0 \tag{42}$$

D. The solution to the following integral is...

$$I = \int\limits_0^\infty (\alpha - \omega)\,\epsilon\,R_0\,\text{Exp}\left\{\omega\,t\right\}\text{Exp}\left\{-\kappa\,t\right\}\delta t = (\alpha - \omega)\,\epsilon\,R_0 \int\limits_0^\infty \text{Exp}\left\{(\omega - \kappa)\,t\right\} \tag{43}$$

Using Appendix Equation (42) above, the solution to Equation (43) above is...

$$I = \frac{\alpha - \omega}{\kappa - \omega}\,\epsilon\,R_0 \text{ ...when... } \omega < \kappa \tag{44}$$